USE OF EDM AND PROPER REDUCTION OF OBSERVATIONS

INTRODUCTION

EDM stands for **E**lectronic **D**istance **M**easurement. There are two basic types of EDM equipment, electro-optical (lightwaves) and electromagnetic (microwaves). In the past ten years or so, the electro-optical instruments have predominated. The emphasis in this workshop will be on electro-optical instruments.

DEFINITIONS

The frequency (f) of a signal is the number of oscillations per second. The wavelength ( )is the length between two successive crests of a sinusoidal wave. The wavelength is equal to the speed of the wave (in this case the speed of light) divided by the frequency.

A meter is equal to 3.280833333 U.S. Survey feet (it is important to carry this computation out to the proper number of significant figures, especially when dealing with state plane coordinates). Temperature is measured in either degrees Fahrenheit or degrees Celsius, and the two are related by:

$$°C = \frac{5}{9} * (°F - 32) \quad 1$$

To convert °C to an absolute scale (Kelvin), add 273.2°. Pressure is measured in inches or mm of mercury (Hg), or in bars. One bar is about 750.06 mm of Hg. Normal atmospheric pressure is 1.01325 bar, 760 mm Hg, or 29.92" Hg.

The standpoint is the station where the instrument is situated, and the forepoint is the point to which observations are being made (backsight or foresights). Figure 1, attached to this paper, shows the various distances discussed. They are described as follows:
d_1 is the wave path length (edm distance corrected for atmospheric delay), d_2 is the wave path chord, d_4 is the ellipsoidal arc length, and d_3 is the ellipsoidal chord length. The horizontal chord length at the standpoint height is denoted d_5. The mark-to-mark distance is d_G. Because of the very small difference between the arc and chord distances, d_{edm} will be used interchangeably with d_1 and d_2, and d_e with d_3 and d_4.

BASIC PRINCIPLES

The basic principle of electro-optical EDM's is the indirect determination of the travel time of a beam of light from the instrument to the reflector and back. The speed of light in a vacuum is well known. However, the measurements surveyors take are of course not in a vacuum. Therefore, we must apply corrections for atmospheric conditions. Because the velocity of light is so great, it is not possible to directly measure the time interval that passes while the light beam travels. To get an accuracy of 0.003 m (0.01 ft), it would be necessary to measure the time interval to an accuracy of $5*10^{-12}$ seconds. To get around this problem, EDM instruments measure the phase difference between the transmitted and received signals. By measuring the phase difference of a single frequency, the fractional part of the distance can be determined very accurately, but the total distance is unknown. By utilizing several different frequencies, the total distance can then be resolved. This phase shift can be determined to an accuracy of about 10^{-3} to 10^{-4}. By using a mean value of $3*10^{-4}$, and a modulation frequency of 14.9854 MHz (corresponding to a unit length of 10 m), the fractional distance can be determined to about 0.003 m. The unit length of 10 m is used in many EDM's today. Exceptions to this are several high precision instruments, such as the Mekometer, which uses a basic unit of 0.3 m, and the Tellurometer MA 100, which uses 2.0 m. These smaller unit lengths are necessary to achieve the higher accuracy. For instance, the phase measurement for a unit length of 0.3 m should be accurate to 0.0001 m.

PROPAGATION THROUGH ATMOSPHERE

As mentioned, the speed of light in a vacuum is a well known constant, having an accepted value of 299,792,458 m/s ±1.2 m/s. The velocity of light in the earth's atmosphere at sea level (under standard conditions) is about 299,702,532 m/s. This is a difference of about 300 ppm (parts per million). This velocity is affected by temperature, pressure, and humidity. The usual practice is to observe the temperature and pressure (for light waves, the humidity is usually ignored) at the instrument and/or reflector. The ratio between the velocity of light in a vacuum (c_0) and the actual velocity (c) is known as the refractive index n, and is computed as follows: $n=c_0/c$. The value mentioned above, at sea level, would be n=1.0003. This value is often converted to the refractive number N, which is computed as follows: $N=(n-1)*10^6$. Therefore, a value of n=1.0003 is equal to N=300. The refractive index depends not only on atmospheric conditions but also on the wavelength.

$$(n_g - 1)*10^{-7} = N_g * 10 = 2876.04 + \frac{3*16.288}{\lambda^2} + \frac{5*0.136}{\lambda^4} \quad 2$$

The group refractive index, n_g is defined for standard dry air at 0°C, 760 mm Hg, and a carbon dioxide content of 0.03%.

The ambient refractive index is as follows:

$$N = \frac{N_g * p - 41.8 * e}{3.709 * T} \quad 3$$

where N_g is the group refractive number, T is the temperature in degrees Kelvin ((add 273.15° to temperature Celsius), p is the pressure in millibars, and e is the partial pressure of water vapor in millibars. To determine how accurately we must measure each of these parameters, we can differentiate this equation as follows:

$$dN_T = \frac{1}{T^2}\left(-\frac{N_g}{3.709} * p + 11.3 * e\right)dT \quad 4$$

$$dN_p = \frac{N_g}{3.709 * T} dP \quad 5$$

$$dN_e = -\frac{11.3}{T} de \quad 6$$

This analysis can be broken down into three distance categories: short (less than 3 km, $\sigma_N=2*10^{-6}$), medium (3 to 12 km, $\sigma_N=1*10^{-6}$), and long (greater than 12 km, $\sigma_N=0.5*10^{-6}$). The vast majority of measurements taken by surveyors fall in the short category.

SHORT DISTANCE: For this analysis, a common value of N=294 will be assumed. A typical instrument used for this type of measurement has an accuracy as stated by the manufacturer of ±(5 mm + 5 ppm). To keep the error due to atmospheric measurements from affecting the accuracy, N should be determined to an accuracy of ± 2 ppm. The following allowable errors were computed: σ_T=±1°C (1.8°F), σ_p=±3 mbar (0.09" Hg), and σ_e=±39.3 mbar (1.18" Hg). For t=30°C, and 90% relative humidity, e=38.2, and it can be seen that the allowable error in e is greater than e itself. So, for short distances, the relative humidity can be ignored. This equation can then be rewritten, ignoring e, in several forms:

$$N = \frac{0.2696 * N_g * p}{273.15 + t} \quad 7$$

$$N = \frac{0.3594 * N_g * p}{273.15 + t} \quad 8$$

$$N = \frac{16.436 * N_g * p}{459.69 + t} \quad 9$$

The first equation is used when p is in millibars, and t is in degrees Celsius, the second equation is for p in mm Hg and t in degrees Celsius, and the third equation is for p in inches Hg and t is in degrees Fahrenheit.

N as computed represents the ppm (parts per million) correction for ambient conditions. However, most instruments have a built-in refractive number N_k. Therefore, the correction to the distance measured by the instrument would be N_k-N (in ppm). As an example, the following is the formula for a Topcon DM-S3 EDM:

$$PPM = 1 + (279.6 - \frac{106.0*p}{273.2 + t}) * 10^{-6} \quad 10 \quad \text{(metric)}$$

$$PPM = 1 + (279.6 - \frac{2692.4*p}{273.2 + \frac{5}{9}*(t - 32)}) * 10^{-6} \quad 11 \quad \text{(feet)}$$

The value 279.6 is the built-in refractive number, and N_g is 295.

The equipment to measure temperature and pressure to this accuracy is small, inexpensive, and commonly available. Some instruments have the capability to enter the temperature and pressure directly into the instrument, and an on-board processor computes and applies the correction. On other instruments, it is necessary to compute the proper ppm correction (using a chart or other device), and enter the ppm correction. Often times only the temperature is measured, and the pressure is computed from the elevation. It is possible to increase the accuracy of this method somewhat by using an inexpensive battery operated weather radio, obtaining the local barometric pressure, and then correcting it. It is important to note that the pressure given in weather reports has been reduced to sea level. It is therefore necessary to correct it for elevation. An approximate formula for computing barometric pressure is:

$$p = \log^{-1} \frac{92670 - elevation}{62737} \quad 12$$

where the elevation is in feet, and the resulting b is in inches of Hg. This can also be used to correct a sea level pressure to actual pressure. The standard procedure is to observe the temperature and pressure at the standpoint only. Better accuracy can be obtained by measuring the conditions at the forepoint also. Temperatures should be measured in the shade, exposed to the wind, and well above the ground. The common practice of laying the thermometer on the ground during the observations and then reading it should be avoided. Care should be taken when observations are being made over steep lines and atmospheric measurements are made at one end only. As mentioned before, the pressure changes in a known manner with respect to changes in elevation. Temperature also changes with elevation, the gradient being approximately 0.0065°C per meter. As an example, for a difference in elevation of 1000 ft (304.8 m), the change in pressure would be 1.08" of Hg (27.4 mm Hg), and the change in temperature would be 3.56°F (1.98°C). For example, at 1000 feet elevation, with a pressure of 28.92" Hg and a temperature of 45°F, the correction would be 1.93 ppm. At 2000 feet elevation, the temperature would be 41.4°F, and the pressure would be 27.88" Hg. This results in a correction of 9.99 ppm. The actual correction applied should be the mean of these two, or 6.0 ppm.As in all survey tasks, a pre-survey analysis is necessary to determine the accuracy required. For short distances, it may be sufficient to simply estimate the temperature and pressure. When high accuracy is required, it is best to enter a value of 0 ppm in the instrument, record the temperature and pressure at both the standpoint and forepoint, and then apply the correction in the office computations. The formula for each instrument is given in the manufacturers manual.

<u>MEDIUM DISTANCES:</u> With the increasing use of GPS, this category
of distances is becoming less common. The discussion above for short distances applies, but the measurement of the atmospheric parameters becomes more critical. The relative humidity can no longer be ignored. A sling psychrometer should be used to determine the temperature and relative humidity. Also, a good meteorological barometer should be used. Measurements should be taken at both ends of the line during the distance measurement. Once again, an accuracy analysis should be performed before the survey to determine the accuracy required. These extra steps should be taken if a survey is being done to first order or second order class I specifications.

<u>LONG DISTANCES:</u> The measurement of long distances requires not only a more careful determination of atmospheric parameters, but also the curvature of the ray path may need to be taken into account:

$$k_1 = -\frac{k^2 * d_1^3}{24 * R^2} \qquad 13 \qquad d_2 = d_1 + k_1 \qquad 14$$

where k is the coefficient of refraction (use 0.13 for light waves), and R is the radius of the spherical earth, approximately equal to 20,906,000 feet. As mentioned in the definitions section, this correction can be safely neglected in most cases. The correction amounts to 0.23 ppm for a distance of 100 km. For a survey of moderate accuracy, the same equipment as for medium distances could be used. For high accuracy surveys, some agencies have gone to great lengths to obtain the proper corrections, such as flying a helicopter or airplane along the ray path during the observations, collecting the atmospheric parameters.
As mentioned, GPS has made the medium and long range measurements with EDM somewhat rare, especially for control surveys.

ERROR SOURCES

There are several types of errors which affect EDM measurements.

The phase measurement is the measurement of the phase shift between the transmitted and received modulation signal. The error in this measurement is random in nature. Because of this, the more measurements taken, the smaller this error becomes. Some instruments have the capability to automatically take several readings. For instance, the Topcon DM-S3 can be programmed to either repeat the measurement, and display each reading separately, or to take a number of readings (i.e. 10), and display the mean and the standard deviation. A cyclic error is also present, which repeats itself over the unit length. In other words, suppose there is an error of 0.002 m in a distance of 702 m, and an error of 0.003 m in a distance of 705 m. These same errors will be present at 752 m and 755 m, for example. It should be mentioned that some older EDM's are susceptible to a 10 m error when the units part of the distance is close to 0 or 10. In other words, a distance of 989.99 might be displayed as 999.99. This is not common, and the newer instruments do not have this problem. One method of preventing its occurrence is to use a "mirror bar". This consists of a metal bar which can be mounted on a tribrach. It has three positions for the reflector, one at zero, and one each at a plus and minus offset. Several sizes have been used, such as a 2 m bar, or a 60 cm bar. The latter has stops at +0.30 m and -0.30 m.

The zero correction or additive constant arises because the electrical center of the instrument does not usually correspond to the physical center of the instrument. The same applies to the reflector. Usually, the manufacturer determines this constant and programs it in to the instrument. However, this value can change with time, and if a different type of reflector is used, the value will change. This value can sometimes be changed in the instrument, or can be applied in the office computations. If a different type of reflector is used, and the distance is not corrected, a significant error can result.

The centering error is self explanatory. This error of course can be reduced by proper adjustment of tribrachs. Note that it can be present at both the standpoint and forepoint, and varies according to whether the error is parallel or perpendicular to the line being measured.

Blunders are not really errors in the sense that they are preventable and are usually of large magnitude. For instance, a distance of 1011.23 may be recorded as 1023.11. This can be avoided by recording the distance in both feet and meters, or by using a data collector which automatically records the data.

A scale error can be caused by several factors. The oscillator in the instrument may have drifted off of the design frequency, or the atmospheric parameters may have been incorrectly determined or entered. The accuracy of the atmospheric correction determination has already been discussed.

REDUCTIONS

The first corrections to be applied to a measured distance are the zero correction and scale differences, as determined by a calibration. These values may be close enough to zero to be omitted, but can easily reach significant amounts, especially when a non-standard prism is being used. Next, the so called velocity corrections are applied. First, the atmospheric correction (also known as the first velocity correction) should be applied to the distance. This will result in the actual distance of the curved ray path. The second velocity correction arises because of the fact that the light ray does not follow a circular path between the standpoint and forepoint. It is more important for microwaves than for light, and for longer distances than short distances. The following examples show its significance for light waves:

Distance	Correction	PPM
1000 m	$2.6*10^{-7}$ m	0.0003
10000 m	0.0003 m	0.0262
100000 m	0.262 m	2.62

As can be seen, it can be ignored for all but high precision long lines.

The next corrections to be applied are the geometric corrections. The distance we have after the velocity corrections is a curved ray between the EDM and the reflector. A very useful distance is the mark-to-mark distance. This distance is independent of ellipsoid or grid used, instrument and reflector heights, and is the best value for archiving a distance. This is the usual output of GPS processing, and should be used when comparing measured distances to GPS vectors. The zenith distances (vertical angles) can also be reduced to mark to mark. The correction to the zenith distance is:

$$\Omega \cong \frac{(h_t - h_{th})*\sin Z_{th}}{d_G}; \quad 15 \qquad Z_G = Z_{th} + \Omega \quad 16 \qquad (\Omega \quad 17 \text{ is in radians})$$

where h_t is the height of the target, h_{th} is the height of the instrument, Z_{th} is the measured zenith distance, and d_G is the mark- to-mark distance. This value is approximate, but is valid if h_t-h_{th} is less than 0.5 m. The mark-to-mark distance is computed as follows:

$$d_G = \frac{d_{edm}}{\sqrt{1 + \frac{(h_{edm} - h_r)^2}{d_G^{\,2}} - \frac{2*(h_{edm} - h_r)}{d_G} * \cos Z_G}} \quad 18$$

Note that d_G is on both sides of this equation, and d_G is necessary to compute Z_G. To do a rigorous computation, it is necessary to perform several iterations. This can easily be programmed into a computer or calculator. However, several approximations are possible. It is possible to use d_{edm} in place of d_G, and Z_{th} in place of Z_G. This will result in an error in d_G of 0.001 m if the heights of instruments, targets, and reflectors are all within 0.5 m of each other, and the distance is greater than 30 m. If elevations are available instead of zenith distances, a simple reduction for d_G is possible:

$$\Delta d = -\left(\frac{(H_2 - H_1)*(h_2 - h_1)}{d_{edm}} + \frac{d_{edm} * h_m}{R}\right); \quad 19 \qquad d_G = d_{edm} + \Delta d \quad 20$$

where H_2 is the elevation of the high point marker, H_1 is the elevation of the low point marker, h_2 is the height of the reflector/instrument at the high point, h_1 is the height of the reflector/instrument at the low point, d_{edm} is the arc length between the standpoint instrument and forepoint reflector (after velocity corrections), $h_m=(h_1+h_2)/2$, and R is the radius of the spherical earth (20,906,000 ft). In all of these reductions, it is assumed that the EDM is collinear with the theodolite, and that the reflector and target are also at the same height. If this condition is not met, the following correction should be applied:

John Hamilton
Geodetic Engineer

hamilton@terrasurv.com
(412) 341-5620

$$d_{th} = d_{edm} + \Delta h * \cos Z_{th} - \frac{\Delta h^2}{2 * d_{edm}} \quad 21 \quad \text{where} \quad \Delta h = h_{edm} - h_{th} + h_t - h_r \quad 22$$

The next correction is the reduction to the ellipsoid. Before NAD 1983, this was commonly called reduction to sea level, since the location of the ellipsoidal surface was not usually known. There are two methods, depending on whether zenith distances or elevations are available. Of course, for zenith distances, it is still necessary to know the elevation of the standpoint, although an approximate value is sufficient. For an accurate reduction, ellipsoidal heights must be used. Here in Pennsylvania, the separation between the GRS 1980 ellipsoid (as used in NAD 1983 and the geoid (approximately sea level) is about 30 m. The ellipsoid is above the geoid. The error caused by ignoring this correction is about 5 ppm (1:200,000). When using zenith distances, reciprocal zenith distances should be used whenever possible.

USING STATION HEIGHTS: To reduce a distance to the ellipsoid, the following procedure is followed. The first correction should be the arc to chord correction, but as mentioned above, it can be safely neglected for all but very long lines. The so called chord to chord correction is:

$$d_e \cong \sqrt{\frac{d_G{}^2 - (H_2 - H_1)^2}{(1 + \frac{H_1}{R}) * (1 + \frac{H_2}{R})}} \quad 23$$

where H_1 is the height of the standpoint above the ellipsoid, H_2 is the height of the forepoint above the ellipsoid, and R is the mean radius of the earth (20,906,000 ft; 6,371,000 m). This reduction is accurate to 1 ppm for distances less than 12 km, and differences in elevation of less than 3000 m. This same formula may be used for d_{edm}, but the h_{th} must be added to H_1, and h_t must be added to H_2. Once again, to be rigorous, the chord distance should be converted to an ellipsoidal arc distance, but this can also be skipped for moderate distances. Alternatively, the following formula goes from d_1 (the arc length) directly to d_4 (the ellipsoidal arc length):

$$d_4 = 2 * R * \arcsin \sqrt{\frac{R^2 * \sin^2\left(\frac{d_1 * k}{2 * R}\right) - \frac{k^2 * (H_2 - H_1)^2}{4}}{k^2 * (R + h_1) * (R + H_2)}} \quad 24$$

This formula is rigorous and contains no approximations. It is important that the difference in elevation between the standpoint and forepoint be accurately known. For a 1000 m distance, with a difference in elevation of 100 m, the difference should be accurate to 0.05 m to attain an accuracy in the distance of 5 ppm (1:200,000). It becomes more critical on steep lines. For the same 1000 m line, but with a difference in elevation of 300 m, it is necessary to know the difference in elevation to 0.016 m.

ZENITH DISTANCE:
When using zenith distances, it is always preferable to use reciprocal observations. A single step method will be shown.

$$d_e = R*\arctan\left(\frac{d_G*\sin\left(Z_G+\varepsilon+\frac{d_G*0.13}{2R}\right)}{R+H_1+d_G*\cos\left(Z_G+\varepsilon+\frac{d_G*0.13}{2R}\right)}\right) \quad 25$$

where Z_1 is the zenith distance (one way), H_1 is the height above ellipsoid for the standpoint. Once again, Z_1 can be substituted for Z_G, and d_{edm} can be substituted for d_G. H_1 would of course include the height of instrument at the standpoint. ε 26 is the deflection of the vertical in the azimuth of the line. This can often be neglected, but in mountainous areas it should be considered. If reciprocal zenith distances are observed, then the refraction correction $\frac{d_G*k}{2R}$ 27 can be eliminated. The angle becomes $(Z_1 + Z_2)/2$.

A refinement for accurate work is to compute the actual radius of curvature of the earth in the azimuth of the line as follows:

$$R_A = \frac{M*N}{M*\sin^2 A + N*\cos^2 A} \quad 28$$

where A is the azimuth of the line, and M and N are computed as follows:

$$M = \frac{a*(1-e^2)}{(1-e^2*\sin^2\phi)^{3/2}} \quad 29$$

$$N = \frac{a^2}{\sqrt{a^2*\cos^2\phi + b^2*\sin^2\phi}} \quad 30$$

where ϕ 31 is the average latitude of the line, and a,b, and e^2 are parameters of the reference ellipsoid (GRS 1980: a=6,378,137 m; b= 6,356,752.314 m; $e^2=(a^2-b^2)/a^2$.

Another computational route is to compute the horizontal distance at elevation H_1, and then reduce it to the ellipsoid. It is important to note that there are an infinite number of horizontal distances between two plumb lines, and it is necessary to specify at which elevation the horizontal distance is computed. The customary method of converting a slope distance (slant range) to a horizontal distance is:

$$HD = SD*\sin(Z) \quad 32$$

There are several problems with this. First, the earth is a curved surface, not flat. Second, the line of sight is refracted, and the zenith distance measured is not the true zenith distance. These problems can be taken care of computationally or by observation methods. To properly compute a horizontal distance, a coefficient of refraction (k) of 0.13 is assumed. However, this coefficient may not reflect the actual conditions, especially when the line is close to the ground. The problem can be eliminated by taking reciprocal zenith distance observations, and using the mean.

The following formula can be used:

$$d_5 = d_{edm}*\sin Z_1 - \frac{d_{edm}^2*(2-k)*\mathrm{SIN}(2*Z_1)}{4R} \quad 33$$

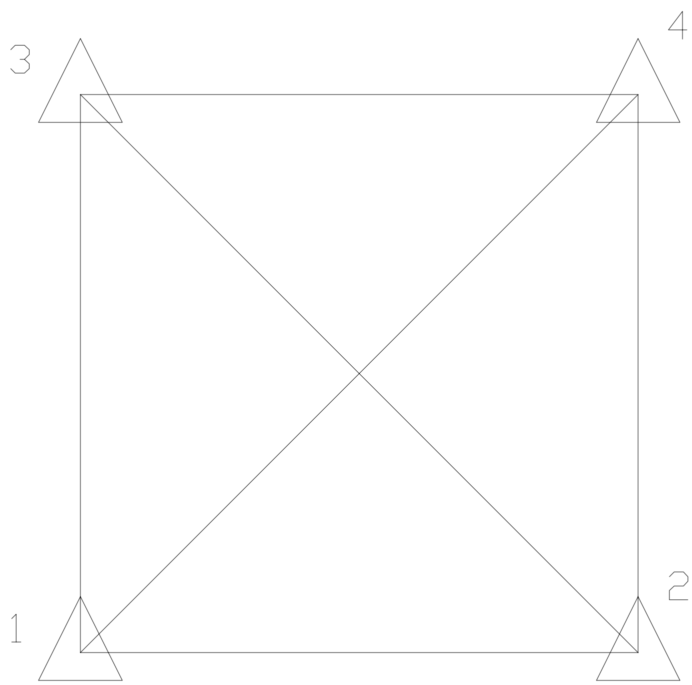
This horizontal distance d_5 can be reduced to d_e by the simple formula:

$$d_e = d_3 = \frac{d_5}{(1 + \frac{H_1}{R})} \quad 34$$

When the distance is going to be used in a grid system, for example the state plane coordinate system, the grid factor should be applied to the ellipsoidal distance, not a "horizontal" distance.

TRILATERATION

Most of the existing geodetic control established in this country by the U. S. Coast and Geodetic Survey (later National Geodetic Survey) was established by triangulation. This is where angles are observed in each triangle in the network. Baselines, or measured distances, were few and far between. With the advent of EDM, trilateration has become feasible. In this method, the lengths of the sides of the triangles are measured. The highest accuracy can be obtained when both angles and lengths are measured. This method, called triangulateration, has been used in deformation surveys, which will be discussed later. One of the advantages of trilateration is that the tedious procedure of observing several (8 to 16) positions of angles is eliminated. A drawback is that careful attention to geometry is required. An example will be given to show the increase in accuracy possible.



A fictitious quadrilateral was analyzed by using the simulation mode of a least squares adjustment package. The input standard deviations for the EDM were ±(5 mm + 5 ppm). This is a typical accuracy for EDM's used by many surveyors. The angles were given a standard deviation of 2 seconds. The stations 1 and 2 were held fixed (known coordinates), and the stations 3 and 4 were to be determined. Four scenarios were tested. The first consisted of pure triangulation. In this method, all eight angles were measured. Next, pure trilateration was used, where the lengths of the six lines were measured (both ways). The third method consisted of a traverse from 1 to 2 to 3 to 4 and closing on 4. Lastly, the method of triangulateration was tested. The following table lists the size of the error ellipses for stations 3 and 4 with respect to the fixed stations 1 and 2 (at the 95% confidence level):

STATION	SEMI-MAJOR AXIS	SEMI-MINOR AXIS	Vertical
Triangulation:			
3	0.038 m	0.029 m	
4	0.038 m	0.029 m	
Trilateration:			
3	0.016 m	0.009 m	0.023 m
4	0.016 m	0.009 m	0.023 m
Traverse:			
3	0.028 m	0.011 m	0.037 m
4	0.028 m	0.011 m	0.037 m
Triangulateration:			
3	0.014 m	0.008 m	0.023 m
4	0.014 m	0.014 m	0.023 m

As expected, the triangulateration is the most accurate. However, there is not much difference between trilateration and triangulateration. The triangulation method gives results which are less accurate by a factor of more than two, and the vertical component is not determined. It can be seen that the tedious process of turning multiple sets of angles hardly affects the final accuracy, whereas the far easier method of distance measurement is the most accurate. In fact, if elevations were available at the points, no angulation (i.e. vertical) would be necessary. When establishing control using triangulateration or trilateration, proper reduction of the observations is very important, as well as careful attention to geometry. Increasing the accuracy of the EDM measurements to 0.001 m ± 2 ppm (and the vertical accuracy to 3") results in the following:

Trilateration:			
3	0.011 m	0.006 m	0.014 m
4	0.011 m	0.006 m	0.014 m

TRIGONOMETRIC LEVELS

The accuracy possible with modern total stations and theodolites makes trigonometric leveling feasible. With special care, it can achieve second order class I vertical accuracy. This has been used by the author for river crossings in a second order class I vertical control network. With hardly any additional effort, it can achieve third order vertical accuracies on a traverse if the sight lengths are kept moderate. When accurate results are desired, care must be taken when tying to bench marks. A very accurate method is to set up a theodolite near a bench mark (less than 30 m away), measure the zenith distances to four different marks on the rod (usually the 1, 2, 3 and 4 foot marks. The difference in elevation between the bench mark and the trunnion axis of the theodolite can be computed as follows:

$$\Delta h = \frac{l_2 * \cot z_1 - l_1 * \cot z_2}{\cot z_1 - \cot z_2} \quad 35$$

where z_1 and z_2 are the measured zenith distances (mean of direct and reverse) to two marks on the rod, and l_1 and l_2 are the readings on the rod. Using four marks gives a total of six determinations, of which the mean can be computed. When measuring the zenith distances on the traverse, a good practice is to measure all three stadia hairs both direct and reverse. All measurements, both zenith distances and EDM distances, should be measured both forward and back. The formula for computing height differences for reciprocal observations is as follows:

$$H_2 - H_1 = \frac{d_{edm}}{2} * (\cos z_{12} - \cos z_{21}) + h_{th} - h_t \quad 36$$

A rigorous formula for the one way computation of height difference is as follows:

$$H_2 - H_1 = d_{edm} * \cos z_1 + (\frac{1-k}{2R}) * (d_{edm} * \sin Z_1)^2 + h_{th} - h_t \quad 37$$

The coefficient of refraction is assumed to have a value of 0.13.

Be aware that k can vary from -2.0 to +1.5 for lines of sight which pass close to the ground. Given a distance of 1500 ft, and a zenith distance of 88°, k=+0.13 gives a correction of 0.047 feet, k=-2.0 gives a correction of 0.161 feet, and k=+1.5 gives a correction of -0.27 feet. This shows the importance of taking near simultaneous reciprocal zenith distance observations.

It is important to note an additional error source. If the zenith distances (the same applies to horizontal angles) are being turned to the prism, and not a target, additional error can be encountered if the prism is not directly facing the instrument. This is especially apparent over short distances, and larger over steep lines. This should be avoided by pointing on a well defined target.

GEODETIC CONTROL SURVEYS

Most geodetic control surveys done today to FGCC specifications are done by GPS. However, this is not to say that a good control survey cannot be performed with a theodolite and EDM. The requirements for length measurements for second order class I (20 ppm, 1:50,000) surveys are as follows: traverse 1:300,000, triangulation (base measurement) 1:900,000, and trilateration 1:750,000. It is for this reason that the above rigorous reductions were given. It is acknowledged that day to day surveying does not usually require these accuracies.

DEFORMATION SURVEYS

A deformation survey is performed to monitor displacement of structures. For example, a dam needs to be monitored to detect movements or settlements which can affect the integrity of the structure. Precise survey methods are usually used along with other instrumentation. To detect vertical movements, precise differential levels are usually run, but trigonometric levels can be used. For horizontal movements, triangulateration is the customary method. In recent years, research has been performed of the feasibility of permanently installing a series of GPS antennas in the structure, and having a constant monitoring method. However, the trilateration method is still probably the most accurate. When performing a survey of this type, the angle measurement is the most tedious and time consuming procedure. It can often be eliminated with proper use of EDM, resulting in a significant savings in cost and time.

An interesting procedure developed by Kenneth D. Robertson of the U.S. Army Corps of Engineers Engineer Topographic Laboratories is the method of Ratios, or reference lines. This method is based on the following observations:

1) Refractive index errors, resulting from end point measurements of temperature and pressure,

tend to be the same for all lines measured from one point within a short period of time.

2) The ratios of observed distances, measured from one point within a short period of time, are constant.

The idea behind this method is to frequently measure the "stable" lines, between pillars, during the measurement of the distances to the axis points. The ratio of the observed distance to the known distance for these reference lines can then be used as a correction to the other lines. Of course, the first time a network is observed, careful atmospheric measurements should be obtained in order to assign "true" values to the reference network. It should also be repeated periodically to ensure that there is no movement of the reference pillars.

REFERENCES

Laurila, Simo H. (1983). *Electronic Surveying in Practice*, John Wiley & Sons Inc., New York.

Robertson, Kenneth D. (1979). *The use and calibration of distance measuring equipment for precise mensuration of dams (Revised)*, U.S. Army Engineer Topographic Laboratories, Fort Belvoir, VA

Rueger, J. M. (1989). *Electronic Distance Measurement*, Springer-Verlag.

Rueger, J. M. (1981). *Practical Results of EDM-Height Traversing*, The Australian Surveyor, Vol. 30, No. 6.

Rueger, J. M. (1982). *EDM-Height Traversing versus Geodetic Leveling*, The Canadian Surveyor, Vol. 36, No. 1.

Krakiwsky, Edward and Vanicek, Petr (1986). *Geodesy: The Concepts*, North-Holland, New York

Torge, Wolfgang (1980). *Geodesy*, Walter de Gruyter, New York

Vicenty, T. (1986). *Geometric Reduction of Measured Lines*, Surveying and Mapping, Vol. 46, No. 3.